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The prediction of maximum forging load and effective stress for different material of bevel gear forging

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Abstract

The manufacture of gears by applying hot or cold bulk forming processes is a quite widespread production method due to its well-known basic advantages such as material and time cost reduction and the increased strength of the teeth. However, the associated process planning and tool design are more complicated. In the precision forging of gears, the workpiece volume, the die design, the power requirement and careful processing are more critical than traditional forging technology. For complete filling up, predicting the power requirement is an important feature of the near net-shape forging process. In this paper, a finite element analysis is utilized to investigate the material properties such as yielding stress, strength coefficient and strain hardening exponent effects on forming load and maximum effective stress. The adductive network was then applied to synthesize the data set obtained from the numerical simulation. The predicted results of the maximum forging load and maximum equivalent stress of bevel gear forging from the prediction model are consistent with the results obtained from FEM simulation quite well. After employing the prediction model one can provide valuable references in prediction of the maximum forging load and maximum equivalent stress of bevel gear forging under a suitable range of material parameters.

Keywords: Bevel gear; Forging; Finite element analysis; Material property; Adductive network

1. Introduction

The precision forging of gears can produce near net-shape forgings with no chipping of their teeth, so it has been used widely in the automobile, astronavigation, etc. The precision of gear forging depends on the precision of the die and its structure, so that the die for precision forging must be designed with consideration of the deformation and the stress state caused by the working pressure and the shrinkage fit. The well-known main advantages of these forgings are: (1) great reduction of material expenses; (2) small machining allowances and close tolerances leading to (3) considerable decrease in machining time and process expenses; and (4) marked increase in strength values due to the favourable microstructure developed in the teeth.

For the forging of bevel gears, the way to complete filling up of the material into a die cavity is regarded as the most important aspect for improving the dimensional accuracy of gears. For complete filling up, predicting the power requirement and improving the dimensional accuracy of the gear are an important feature of the forging process. Computer aided engineering (CAE) techniques have been increasingly applied with great success in metal-forming research to predict the forming load, stress and strain distribution. Meidert et al. [1] proposed two modeling

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techniques, finite element (FE) based numerical modeling and physical modeling with plastic, are being presented as process design tools in cold forging. Mamalis et al. [2] used the implicit FE code MARC and the explicit FE code DYNA 3D to simulate the bevel gear forging process. The simulation results by MARC seem to be in good agreement with the experimental results and, therefore, they enable the forging designer to easily create a CAD/CAM/CAE system for analysing the precision-forging problem successfully. Contrarily, the explicit FE code DYNA 3D seems to fail to simulate the whole problem at a very early stage of the analysis due to structural limitations of the code. Recently, Yang [3] used FEM software DEFORM-3D to simulate the spur gear forging process. The load predicted by the DEFORM-3D is closer to the experimental data than the prediction by Choi et al. [4]. Thus, the DEFORM-3D is appropriate to simulate the forging process of gear. It is necessary to perform a lot of numerical simulations obtain a suitable range of the process or material parameters for producing an acceptable product in metal forming process. Lin and Kwan [5] used the finite element method in conjunction with adductive network to predict an acceptable product of which the minimum wall thickness and the protrusion height fulfil the industrial demand on the T-shape tube hydroforming process. Yang and Hsu [6] used a finite element analysis investigate the maximum forging force and final face width under different process parameters such as modules, number of teeth, and the ratio of the height to diameter of billet. The adductive network is then applied to synthesize the data set obtained from the numerical simulation, and a prediction model is established ultimately.

In this paper, a finite element analysis is utilized to investigate the material properties such as yielding stress, strength coefficient and strain hardening exponent effects on forming load and maximum effective stress. The adductive network was then applied to synthesize the data set obtained from the numerical simulation. The predicted results of the maximum forging load and maximum equivalent stress of bevel gear forging from the prediction model are consistent with the results obtained from FEM simulation quite well. After employing the predictive model can provide valuable references in prediction of the maximum forging load and maximum equivalent stress of bevel gear forging under a suitable range of material parameters.

2. Basic theory

2.1 Finite element modeling

The finite element method has been applied to simulate the plastic flow of metal materials during the forming process. For the bevel gear forging process of a plastic deformation problem, the governing equations for the solution of the mechanics in plastic deformation for metal materials involve equilibrium equations, yield criterion, constitutive equations and compatibility conditions. The duality of the boundary value problem and the variation problem can be seen clearly by considering the construction of the function [7]:

$$\pi = \int_v \bar{\sigma}\dot{\bar{\varepsilon}}\, dv - \int_s F_i u_i\, ds \tag{1}$$

where $\bar{\sigma}$ is the effective stress, $\dot{\bar{\varepsilon}}$ is the effective strain-rate, F_i represents the surface tractions and, u_i is the velocity components. The variational form for finite-element discretization is given by:

$$\delta\pi = \int_v \bar{\sigma}\delta\dot{\bar{\varepsilon}}\, dv + k\int_v \dot{\varepsilon}_v \delta\dot{\varepsilon}_v\, dv - \int_s F_i \delta u_i\, ds = 0 \tag{2}$$

where $\dot{\varepsilon}_V = \dot{\varepsilon}_{ii}$ is the volumetric strain rate, π is functional of the total energy and work, and k, a penalty constant, is a very large positive constant. $\delta\dot{\bar{\varepsilon}}$ and $\delta\dot{\varepsilon}_v$ are the variations in effective strain rate and volumetric strain rate. Eq. (1) and Eq. (2) are the basic equation for the finite element formulation.

A commercial FE code DEFORM-3D [8] is adopted to analyze the plastic deformation of the near net-shape bevel gear forging from a sintered metal billet. The iteration methods adopted for solving the nonlinear equations are Newton-Raphson and the direct iteration methods. The direct iteration method is used to generate a good initial guess for Newton-Raphson method, whereas Newton-Raphson method is used for speedy final convergence. The convergence criteria for the iteration are the velocity error norm $\|\Delta v\|/\|v\| \le 0.01$ and the force error norm $\|\Delta F\|/\|F\| \le 0.1$, where $\|v\|$ is defined as $\left(v^T v\right)^{1/2}$.

2.2 Adductive network synthesis and evaluation

In the abductive network, a complex system can be decomposed into smaller, simpler subsystems grouped into several layers using polynomial functional nodes. The polynomial network proposed by Ivakhnenko [9] is a group method of data handling (GMDH) techniques. Theses nodes evaluate the limited number of inputs by a polynomial function and generate an output to serve as an input to subsequent nodes of the next layer. The structure of polynomial network is shown in Fig. 1 [10]. It consists of sigma (summation) units in the hidden layer and pi (product) units in the output layer. Output of a sigma unit is a weighted sum of its inputs, and output of a pi unit is a product of its input. Let the k^{th} input pattern to the network be specified by $X_k = [x_{0k}, x_{1k}, x_{2k}, \ldots, x_{nk}]$, and let the weight associated with connection from input unit i to hidden unit j be w_{ij}. Then, the output z_{jk} of the j^{th} sigma unit is given by

$$z_{jk} = \sum_{i=0}^{n} w_{ij} x_{ik} \tag{3}$$

and output y_k of the network is given by

$$y_k = \prod_{j=1}^{h} z_{jk} \tag{4}$$

where h is the number of hidden units in the network. Combining Eqs. (4) and (5), the general polynomial function in a polynomial functional node can be expressed as:

$$y_k = c_0 + \sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} c_{ijk} x_i x_j x_k + \cdots \tag{5}$$

where x_i, x_j, x_k are the inputs, y_k is the output and c_i, c_{ij}, c_{ijk} are the coefficients of the polynomial functional nodes. In the present study, several types of polynomial nodes are used in polynomial network for predicting the maximum forging force and final face width under a suitable range of process parameters. More detailed explanation of these polynomial functional nodes is available the paper of Ivakhnenko [9].

To build a complete adductive network, the first requirement is to train the database. The information given by the input and output parameters must be sufficient. A predictive square error (*PSE*) criterion [11] is then used to automatically determine an optimal structure. The principle of the *PSE* criterion is to select the least complex yet still accurate network as possible. The *PSE* is composed of two terms, that is:

$$PSE = FSE + K_p \tag{6}$$

where *FSE* is the average square error of the network for fitting the training data and K_p is the complex penalty of the network, shown as the following equation:

$$K_p = CPM \frac{2\sigma_p^2 K}{N} \tag{7}$$

where *CPM* is the complex penalty multiplier, K is a coefficient of the network, N is the number of training data to be used and is a prior estimate of the model error variance.

3. Results and discussion

A schematic diagram of the bevel gear forging process is shown in Fig. 2. The material is upset by the punch. As a consequence the billet material flows into the teeth region. Yang [6] showed the DEFORM-3D is appropriate to simulate the forging process of gear. Thus, the FEM software DEFORM-3D is used in the present simulation for bevel gear forging. During the analysis, the die and the punch are assumed to be rigid. The flow stress for the billet is expressed as Ludwik's form:

$$\dot{\sigma} = Y_0 + c\varepsilon^n \tag{8}$$

where σ is the effective stress, ε is the effective strain, Y_0 is the yielding stress, c is the strength coefficient and n is the strain hardening exponent of material. The modulus of bevel gear m is 0.3, the teeth number of bevel gear N is 12 and the length of bevel gear H as shown in Fig. 2, is 12 mm. The constant shear friction factor is assumed to be 0.01 at the billet/punch and billet/die interfaces, and the punch velocity V_p is 0.075 mm/s in this study. Fig. 3 shows the effective stress and strain distribution for the yielding stress of 100 MPa, strength coefficient of 100 MPa and strain hardening exponent of 0.5. The effective stress and effective strain near the tip of

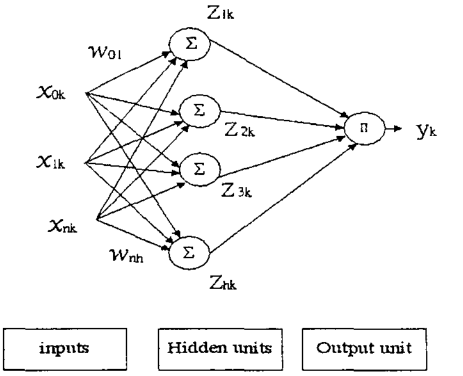


Fig. 1. Structure of polynomial network [10].

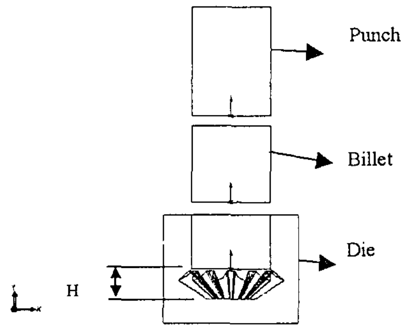
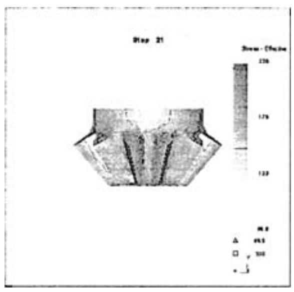
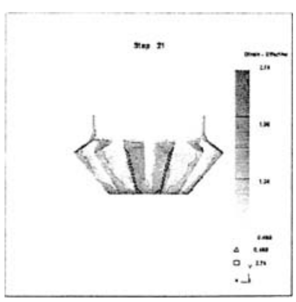


Fig. 2. Schematic diagram of the bevel gear forging process.



(a) Effective stress distribution



(b) Effective strain distribution

Fig. 3. Effective stress and strain distribution of bevel gear forging.

teeth is greater than other region of the bevel gear. The maximum effective stress and the maximum effective strain are 220 MPa and 2.74, respectively.

3.1 Effects of the material parameters on the forging load and maximum effective stress

To investigate the effects of material parameters such as the yielding stress Y_0, strength coefficient c and strain-hardening exponent n of the material on the forging load and maximum effective stress of the bevel gear forging. Numerical analysis was performed for various values. Figs. 4, 5 and Table 1 show the effect of the yielding stress on the forging load and maximum effective stress. Larger values of the yielding stress result in higher values of the maximum forging load and maximum effective stress of the bevel gear forging. Figs. 6, 7 and Table 1 show the effect of the strength coefficient on the forging

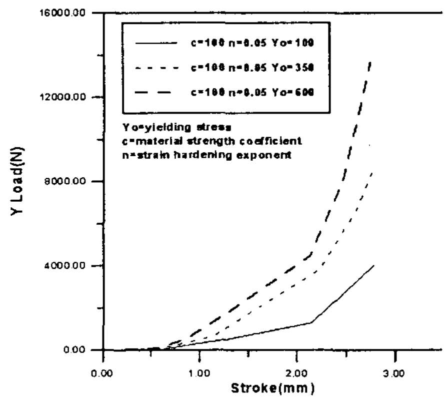


Fig. 4. Effect of yielding stress on the punch load in the bevel gear forging.

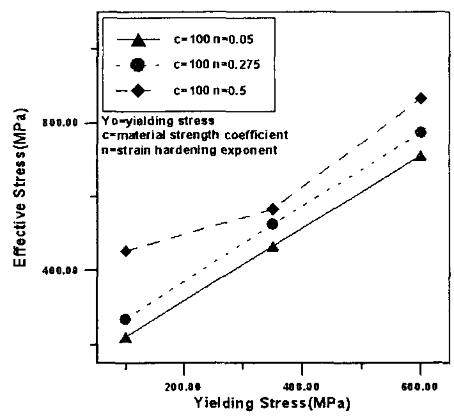


Fig. 5. Effect of yielding stress on the effective stress of billet in the bevel gear forging.

load and maximum effective stress of the bevel gear forging. The maximum forging load and maximum effective stress increases with increasing the strength coefficient of the billet. Figs. 8, 9 and Table 1 show the effect of the strain hardening exponent on the forging load and maximum effective stress of the bevel gear forging. Larger values of the strain hardening exponent result in greater value of the maximum forging load and maximum effective stress.

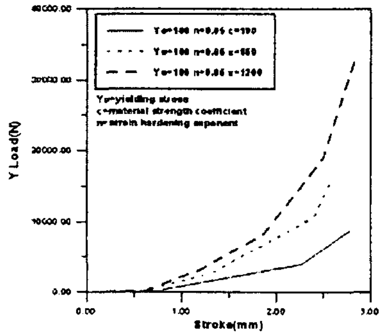


Fig. 6. Effect of strength coefficient on the punch load in the bevel gear forging.

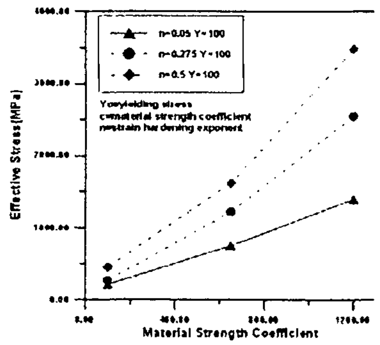


Fig. 7. Effect of strength coefficient on the effective stress of billet in the bevel gear forging.

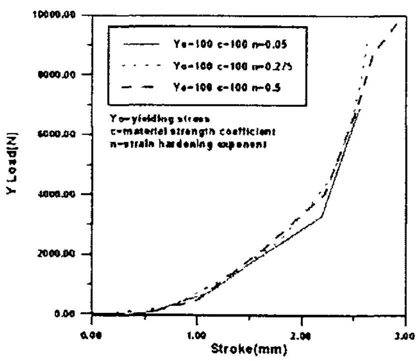


Fig. 8. Effect of strain hardening exponent on the punch load in the bevel gear forging.

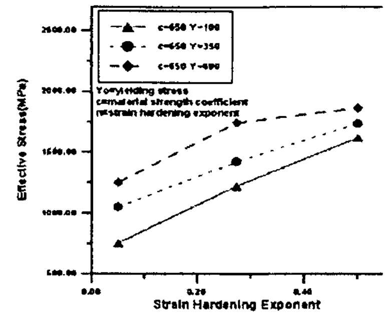


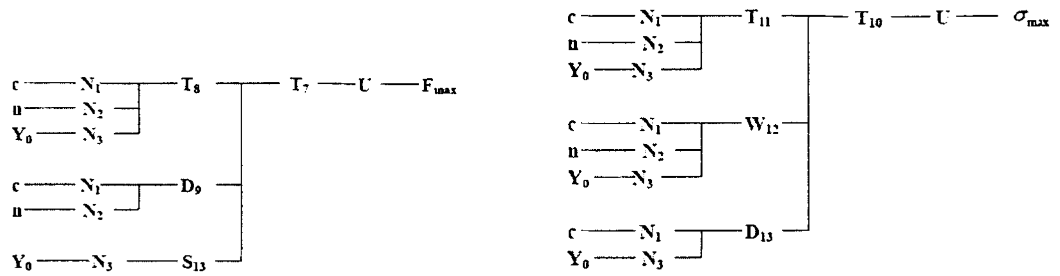
Fig. 9. Effect of strain hardening exponent on the effective stress of billet in the bevel gear forging.

3.2 The prediction model for maximum forging load and effective stress

The yielding stress Y_0 is varied in the range of 100-600 MPa, while the other material parameters are selected by varying the strength coefficient c and the strain hardening exponent n in the ranges of 100-1200 MPa and 0.05-0.5, respectively. There are three process variables and each of these variables was set at three levels. Therefore, 27 (3×3×3) combinations of material parameters are constituted totally and are shown in Table 1. Based on the training database regarding to material parameters, the maximum forging load and maximum equivalent stress of the bevel gear forging shown in Table 1, the adductive

Table 1. Effect of material parameters on maximum forging force and maximum effective stress.

c	n	Y_0	F_{max} (N)	σ_{max} (MPa)
100	0.05	100	4540	212
100	0.05	350	8740	466
100	0.05	600	14800	711
100	0.275	100	4970	268
100	0.275	350	8430	525
100	0.275	600	10200	773
100	0.5	100	7400	453
100	0.5	350	10500	566
100	0.5	600	16100	866
650	0.05	100	17500	752
650	0.05	350	20800	1050
650	0.05	600	23400	1250
650	0.275	100	21000	1220
650	0.275	350	24800	1420
650	0.275	600	26700	1740
650	0.5	100	36300	1620
650	0.5	350	37630	1735
650	0.5	600	38560	1864
1200	0.05	100	27500	1400
1200	0.05	350	32700	1660
1200	0.05	600	38900	1900
1200	0.275	100	48900	2550
1200	0.275	350	49800	2710
1200	0.275	600	49900	2800
1200	0.5	100	64900	3480
1200	0.5	350	74900	3630
1200	0.5	600	86100	3840



(a) prediction model for maximum forging load (b) prediction model for maximum effective stress

Fig. 10. Abductive networks built for predicting the maximum forging force and maximum effective stress in of bevel gear forging.

Table 2. Comparison of the maximum forging force and maximum effective stress between the abductive network prediction and FEM of the bevel gear forging.

Y_0 (MPa)	C (MPa)	n	F_{max} by predicted model (N)	F_{max} by FEM (N)	Error (%)	σ_{max} by predicted model(MPa)	σ_{max} by FEM (MPa)	Error (%)
400	800	0.125	24679	26556	7.07	1436.3	1533	6.31
300	900	0.125	24364	26327	7.46	1494.3	1659	9.92
450	800	0.2	27914	29588	5.66	1652.2	1807	8.56
475	850	0.2	29887	31747	5.86	1755	1853	5.28

networks with a criterion of minimum square error can be developed for predicting the maximum forging load and maximum equivalent stress of the bevel gear forging under a suitable range of material parameters. Two networks shown in Fig. 10 are built for predicting the maximum forging load and maximum equivalent stress of bevel gear forging. All of the associated polynominal equations corresponding to these networks are listed in Appendix 1. The predicted square errors (PSE) in Eq. (6) are 0.00429, and 0.00927 for the prediction of the maximum forging load and maximum equivalent stress of bevel gear forging, respectively

In order to validate the accuracy of the prediction model, another 4 data sets of the suitable range are tested for the maximum forging load and maximum equivalent stress of bevel gear forging predictions. Table 2 shows the comparison of the maximum forging load and maximum equivalent stress of bevel gear forging between the adductive network prediction and FEM simulation under various combinations of material parameters, which are around the border of suitable range. The predicted results of the maximum forging load and maximum equivalent stress of bevel gear forging are consistent with those obtained from FEM simulations quite well. Therefore, the developed networks have a reasonable accuracy for modelling of the bevel gear forging process for determining the maximum forging load and maximum equivalent stress of bevel gear forging.

4. Conclusions

In this paper, a prediction model has been established for determining the maximum forging load and the maximum effective stress of bevel gear forging by using the finite element method (FEM) in conjunction with an adductive network. Different yielding stress, strength coefficient and strain hardening exponent were taken into account as the material parameters in this study.

The influences of the material parameters such as the yielding stress, strength coefficient and strain hardening exponent on the the maximum forging load and the maximum effective stress of bevel gear forging are also examined. The abductive network was then applied to synthesize the data sets obtained from the numerical simulation. The predicted results of the maximum forging load and the maximum effective stress from the prediction model are in good agreement with the results obtained from the FEM simulation. By employing the predictive model, it can provide valuable references to the prediction of the maximum forging load and the maximum effective stress under a suitable range of process parameters in bevel gear forging.

Acknowledgment

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Appendix 1

N_1 : -1.42 + 0.00219c

N_2 : -1.42 + 5.34n

N_3 : -1.68+0.00481Y_0

T_8 : $-0.238+0.831N_1+0.393N_2+0.154N_3+0.118N_1^2+128N_2^2+0.00191N_3^2+0323N_1N_2+0.0252N_1N_3+0.0123N_2N_3+0.0379N_1N_2N_3$

D_9 : $-0.237+0.831N_1+0.393N_2+0.118N_1^2+0.128N_2^2+0.323N_1N_2$

T_7 : $-0.0181-3.1T_8+4.06D_9+0.118T_8^2+0.128D_9^2+323T_8D_9-0.0481S_{13}^2+10.5T_8D_9+1.28T_8S_{13}-0.994D_9S_{13}-0.272T_8D_9S_{13}+0.696T_8^3-0.68D_9^3$

F_{max} : $2.99e4+2.16e4T_7$

T_{11} : $-0.111+0.849N_1+0.84N_2+0.168N_3+0.129N_1^2-0206N_2^2+0.00646N_3^2+0.304N_1N_2-0.017N_1N_3-0.0266N_2N_3-0.00373N_1N_2N_3$

W_{12} : $0.849N_1+0.384N_2+0.168N_3$

D_{13} : $-0.131+0.849N_1+0.168N_2+0.129N_1^2+0.00646N_2^2$

T_{10} : $0.0311+0.725T_{11}-0.0778W_{12}+0.329D_{13}-0.657T_{11}^2-0913W_{12}^2-0.00652D_{13}^2+0.585T_{11}W_{12}+15T_{11}D_{13}-1.05W_{12}D_{13}-0.185T_{11}W_{12}D_{13}+0.171T_{11}^3-0.0668W_{12}^3+0.0793D_{13}^3$

σ_{max} : $1.54e3+1.04e3T_{10}$

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